

# **Validating Life-Cycle Models: Lifetime Earnings and the Timing of Retirement**

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This paper provides a method for validating, and distinguishing between, different life-cycle models of consumption and labor supply. We derive qualitative predictions for two prominent life-cycle model formulations – having non-separable and separable in consumption and leisure – and we use data on the relationship between lifetime earnings and the timing of retirement to assess and distinguish between them. Preliminary estimates provide some support for the non-separable formulation.

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# Validating Life-Cycle Models: Lifetime Earnings and the Timing of Retirement\*

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# Validating Life-Cycle Models: Lifetime Earnings and the Timing of Retirement

## 1. Introduction

The life-cycle model is central to contemporary economic analysis of saving and retirement behavior, and related public policies. This paper attempts to use HRS data on household earnings and retirement ages to discriminate among several prominent formulations of the model. Our approach starts from a theoretical framework with dynamic maximization, but it seeks to estimate linear equations derived from first-order approximations. Our hope is that this estimation strategy will provide versatility in determining which elements are ultimately the most essential for a simple, yet useful, structural model.

This paper builds from a general first-order restriction for optimal household retirement based on a so-called “free endpoint” condition from optimal control theory (see Laitner and Silverman [2007]). This condition may be adapted to a wide variety of life-cycle models and implies that, when choosing its best retirement age, a household balances its loss of current earnings, converted to units of utility, against its utility gain from retirement. It generates restrictions on the parameters of our linear regression equations.

This paper focuses on two prominent life-cycle formulations. Formulation I adopts an intratemporally non-separable utility framework (Auerbach and Kotlikoff [1987], Altig et al. [2001], French [2005], Laitner and Silverman [2005, 2007], Cooley and Prescott [1995]); Formulation II assumes that a household’s utility is additively separable in consumption expenditure and leisure (Bound [1998], Rust and Phelan [1997], Gustman and Steinmeier [1986, 2000]). While differences between Formulations I-II may at first seem modest, they have potentially important implications for behavior and policy.

For example, Formulation I is inherently consistent with an absence of time trend in average retirement age (King et al. [1988]) and, correspondingly, an absence of correlation between retirement age and earnings in the cross section. We see below that Formulation II, in contrast, generally positively associates earlier retirement and higher earnings. Formulation I also tends to predict a routine drop in consumption expenditures for households at retirement (Hurd and Rohwedder [2003], Laitner and Silverman [2005]), whereas Formulation II does not. Empirical evidence for each phenomenon (i.e., lack of trend in the timing of retirement, and routine decline in consumption at retirement) remains controversial; however, the potential importance of each — especially, perhaps, the first — is great.

At this stage, our tentative conclusion is that Formulation I, with non-separable utility, receives some support from our cross-sectional data. Section 5 suggests steps for continuing research.

## 2. Basic Framework

We begin with a basic life-cycle framework that can accommodate both the non-separable and separable utility functions. Then we derive our basic optimality condition for retirement for each formulation and, from the condition, derive sign restrictions for the coefficients of a regression equation for retirement ages. The regression equation, in turn, is the basis for the paper’s empirical analysis.

For expositional convenience, begin with a single member household. The household chooses (i) how much to consume at each age and (ii) the age at which it will retire. Assume that work options are discrete: employers require full-time work; to obtain reduced work hours, an individual must retire altogether. The household starts at age and date 0, lives to age  $T$ , and, when not retired, has earnings flow  $y_t$ . The household solves

$$\max_{c_t, R} \int_0^R e^{-\rho \cdot t} \cdot u(c_t, t, R) dt + \varphi(a_R, R) \quad (1)$$

$$\text{subject to: } \dot{a}_t = r \cdot a_t + y_t - c_t,$$

$$a_0 = 0,$$

where the function  $\varphi(a_R, R)$  gives post-retirement utility conditional on retirement age  $R$  and satisfies

$$\varphi(A, R) \equiv \max_{c_t} \int_R^T e^{-\rho \cdot t} \cdot u(c_t, t, R) dt \quad (2)$$

$$\text{subject to: } \dot{a}_t = r \cdot a_t - c_t,$$

$$a_T \geq 0,$$

$$a_R = A.$$

**Formulation I.** Consider the non-separable formulation first. In Formulation I, for some  $\gamma < 1$  and  $\lambda > 1$  our household's flow utility satisfies

$$u(c, t, R) \equiv \begin{cases} \frac{1}{\gamma} \cdot [c]^\gamma, & \text{if } t < R, \\ \frac{1}{\gamma} \cdot [\lambda \cdot c]^\gamma, & \text{if } t \geq R. \end{cases} \quad (3)$$

A household thus enjoys an improved utility function after retirement as every level of consumption generates a higher level of utility. Indeed, within this formulation it is this improvement that causes an agent to retire in the first place.

Discussion One possible motivation for the non-separable formulation follows Laitner and Silverman [2005, 2007]. In this case, a Cobb–Douglas neoclassical production function  $f$  takes market consumption and leisure as inputs and generates a service flow<sup>1</sup>

$$f(c, \ell) \equiv [c]^\alpha \cdot [\ell]^{1-\alpha}, \quad \alpha \in (0, 1);$$

and, flow utility is an isoelastic function of the flow of services<sup>2</sup>

$$u_t = \frac{1}{\bar{\gamma}} \cdot [f(c, \ell)]^{\bar{\gamma}}.$$

If we normalize post–retirement leisure to 1 and denote pre–retirement leisure by  $\bar{\ell} \in (0, 1)$ , then specification (3) emerges when

$$\alpha \cdot \bar{\gamma} = \gamma \quad \text{and} \quad \lambda = 1/[\bar{\ell}]^{(1-\alpha)/\alpha}.$$

Leisure and consumption are complementary in any neoclassical production function — providing an intuition for the more favorable utility flow in (3) after retirement.

A second motivation for (3) would be that home–production efficiency improves upon retirement from formal work responsibilities.<sup>3</sup> For example, studying the American Time Use Survey 2003, Hamermesh [2005] writes,

There may be fixed time costs of market work such that the effectiveness of the remaining time devoted to ST [secondary and tertiary activities] and L [leisure] is reduced by constant fractions  $\mu_{ST}$  and  $\mu_L$  when even a small amount of market work is undertaken. The fixed time costs might, for example, stem from a need to hurry in one’s other activities (e.g., racing through one’s breakfast in order to get to work on time, foregoing watching *The Tonight Show* in order to be rested for work the next morning). They might induce workers to engage in a different, and perhaps less satisfying mix of other activities .... [p. L-5 and L-6]

Similarly, upon retirement a household may be free to relocate to a locale more efficient for consumption and leisure activities and free to schedule its consumption to take advantage of non–peak–load prices.

A third motivation would argue that employment generates expenses for clothing, entertainment, transportation, and residential location that diminish upon retirement. Provided these expenses are proportional to a household’s living standard, one could interpret (3) as their manifestation.

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<sup>1</sup> French [2005], Cooley and Prescott [1995], and Laitner and Silverman [2005, 2007] use a Cobb–Douglas production function. Auerbach and Kotlikoff [1987] and Altig et al. [2001] use a CES function. The latter includes the Cobb–Douglas as a special case.

<sup>2</sup> Virtually all of the existing literature employs the isoelastic functional form for utility.

<sup>3</sup> See also House, Laitner, and Stolyarov [2007] and Aguiar and Hurst [2005].

Consumption and Retirement Given Formulation I, Laitner and Silverman [2007] shows that the solution to (1)-(3) satisfies the following equations:

$$[\lambda]^{\frac{\gamma}{1-\gamma}} \cdot c_{R-} = c_{R+}, \quad (4)$$

$$c_t = \begin{cases} c_0 \cdot e^{\frac{r-\rho}{1-\gamma} \cdot t}, & \text{if } t < R, \\ [\lambda]^{\frac{\gamma}{1-\gamma}} \cdot c_0 \cdot e^{\frac{r-\rho}{1-\gamma} \cdot t}, & \text{if } t \geq R, \end{cases} \quad (5)$$

where  $c_{R-}$  is the optimal rate of consumption the moment before retirement, and  $c_{R+}$  is the optimal rate just after  $R$ . Solution of (1)-(3) also yields a first-order condition for  $R$  — a so-called “free endpoint condition” — indicating that at the optimal retirement age  $R$ ,

$$[c_{R-}]^{\gamma-1} \cdot [y_{R-} - c_{R-} + c_{R+}] = \frac{1}{\gamma} \cdot [\lambda \cdot c_{R+}]^{\gamma} - \frac{1}{\gamma} \cdot [c_{R-}]^{\gamma}. \quad (6)$$

Combining (4) and (6), at the household’s optimal retirement age  $R$  one has

$$\frac{y_{R-}}{c_{R-}} = 1 + \lambda^{\frac{\gamma}{1-\gamma}} + \frac{1}{\gamma} \cdot [\lambda^{\frac{\gamma}{1-\gamma}} - 1], \quad (7)$$

where  $y_{R-}$  is the household’s earning flow at the moment before retirement. The left-hand side of (7) measures household earnings at the moment just prior to retirement, converted to units of utility with the costate variable from optimal control theory, and the right-hand side gives the utility gain upon retirement. If at a candidate retirement age  $R$  the left-hand side of (7) is greater (smaller) than the right-hand side, the household should postpone (advance) its retirement.

Defining

$$F(R) \equiv \frac{y_{R-}}{c_{R-}} - \left\{ 1 + \lambda^{\frac{\gamma}{1-\gamma}} + \frac{1}{\gamma} \cdot [\lambda^{\frac{\gamma}{1-\gamma}} - 1] \right\}, \quad (8)$$

at optimal retirement age  $R^*$  we have

$$F(R^*) = 0 \quad \text{and} \quad F'(R^*) < 0. \quad (9)$$

To characterize  $F(\cdot)$  in more detail, define

$$\sigma \equiv \frac{r-\rho}{1-\gamma}, \quad \Lambda \equiv \lambda^{\frac{\gamma}{1-\gamma}}, \quad Y = Y(R) \equiv \int_0^R e^{-r \cdot s} \cdot y_s \, ds.$$

Then (5) and budgetary accounting imply

$$c_{R-} = c_0 \cdot e^{\sigma \cdot R} \quad \text{and} \quad c_0 = c_0(R) \equiv \frac{Y(R)}{\int_0^R e^{-r \cdot s} \cdot e^{\sigma \cdot s} \, ds + \Lambda \cdot \int_R^T e^{-r \cdot s} \cdot e^{\sigma \cdot s} \, ds}. \quad (9)$$

So, for the first term on the right-hand side of (8) we have

$$\ln(y_{R-}/c_{R-}) = \ln(y_{R-}/Y(R)) - R \cdot \sigma + \ln\left(\int_0^R e^{-r \cdot s} \cdot e^{\sigma \cdot s} ds + \Lambda \cdot \int_R^T e^{-r \cdot s} \cdot e^{\sigma \cdot s} ds\right). \quad (10)$$

The second right-hand side term is a constant.

Married Households with Children To make better use of our data, we would like to generalize the analysis above to accommodate married households with children. (Indeed, the empirical analysis in this paper is limited to couples.) To do so, let  $R$  now denote the male's retirement age, and let household earnings be the sum of male and female earnings,  $y_s = y_s^m + y_s^f$ . We allow a spouse and children to affect consumption as in Tobin [1967], where "adult equivalent" weights are determined.

We model male and female earnings with separate earnings dynamics equations

$$\ln(y_{it}^m) = X_{it}^m \cdot \beta^m + \mu_i^m + \epsilon_{it}^m \quad \text{and} \quad \ln(y_{it}^f) = X_{it}^f \cdot \beta^f + \mu_i^f + \epsilon_{it}^f, \quad (11)$$

where  $X_{it}$  contains a quadratic in experience and time dummies (reflecting technological progress); the first error component,  $\mu_i$ , is a random individual effect;  $\epsilon_{it}^m$  and  $\epsilon_{it}^f$  are (normally distributed) errors that are independent of each other and of the individual effects; and the vector  $(\mu_i^m, \mu_i^f)$  is bivariate normal. For the first term on the right-hand side of (10), we need to sum male and female earnings in the numerator and denominator. Disregarding the purely random terms, use (11) to define  $\psi(\cdot)$  with

$$\psi(R, \mu^m, \mu^f) \equiv \ln\left(\frac{y_R^m + y_R^f}{Y_R^m + Y_R^f}\right). \quad (12)$$

A combination of (7), (8), (10), and (12) defines a bivariate function  $\Psi(\cdot)$ :

$$F(R) \equiv \Psi(R, \psi(R, \mu^m, \mu^f)). \quad (13)$$

Conditions (7) and (9) yield

$$\begin{aligned} \Psi(R^*, \psi(R^*, \mu^m, \mu^f)) &= 0 \quad \text{and} \\ \frac{\partial \Psi(R^*, \psi(R^*, \mu^m, \mu^f))}{\partial R} + \frac{\partial \Psi}{\partial \psi} \cdot \frac{\partial \psi(R^*, \mu^m, \mu^f)}{\partial R} &< 0. \end{aligned} \quad (14)$$

From (10),

$$\frac{\partial \Psi}{\partial \psi} = 1. \quad (15)$$

The implicit function theorem and the first part of (14) determine the optimal male retirement age  $R^*$  as a function of the individual heterogeneity in lifetime earning ability  $\mu^m$  and  $\mu^f$ :

$$R_i^* = \Xi(\mu_i^m, \mu_i^f, \text{other variables}).$$

A first-order Taylor approximation of  $\Xi(\cdot)$  (with the partial derivatives of  $\Xi(\cdot)$  evaluated at mean values of their arguments) gives us a regression equation relating variation in the timing of retirement to, most importantly, variation in earnings. We divide our sample of households into 3 parts: a set N in which the wife never works; a set S in which the wife works at some ages but stops prior to her husband's retirement; and, the residual set O of "other" households. For simplicity, in the last case — which we think of as the base case (e.g., Gustman and Steinmeier [2000]) — we assume the couple retires together. Our retirement regression equation is

$$R_i = Z_i \cdot \xi + \mu_i^m \cdot \chi_i^N \cdot \alpha_m^N + \mu_i^m \cdot \chi_i \cdot \alpha_m + \mu_i^f \cdot \chi_i \cdot \alpha_f + \mu_i^m \cdot \chi_i^S \cdot \alpha_m^S + \mu_i^f \cdot \chi_i^S \cdot \alpha_f^S + \eta_i, \quad (16)$$

where  $R_i$  is the observed male retirement age for household  $i$ ;  $Z_i$  contains the "other variables;"  $\xi$  is a vector of parameters to be estimated;  $\chi_i^N$  is an indicator that is 1 if the wife in household  $i$  never worked and 0 otherwise, and  $\alpha_m^N$  is the coefficient on male earnings to be estimated in this case;  $\chi_i$  is an indicator that is 1 if both the husband and wife work until  $R_i$  and 0 otherwise, and  $\alpha_m$  and  $\alpha_f$  are parameters to be estimated; and,  $\chi_i^S$  is an indicator that is 1 if the wife works in youth but retires (more than a year) prior to her husband and 0 otherwise, and  $\alpha_m^S$  and  $\alpha_f^S$  are parameters to be estimated. The regression error is  $\eta_i$ .

Interpretation of Regression Equation (16) The conceptual framework underlying our estimation of (16) may be interpreted as follows. As a man and woman marry, their earning abilities  $\mu_i^m$  and  $\mu_i^f$  are known. The couple makes its plans for lifetime consumption, asset accumulation/decumulation, labor supply, etc. Equation (7) should hold prospectively at the beginning of marriage.

Constraints on Parameters To test the validity of our non-separable formulation of the life-cycle model, we derive *a priori* restrictions on the alpha parameters of regression equation (16). Let sets N, S, and O be as above.

For households in N,  $\alpha_m^N$  in (16) corresponds to  $\partial R_i^* / \partial \mu_i^m$ . Using the implicit function theorem on the first part of (14),

$$\frac{\partial R_i^*}{\partial \mu_i^m} = A \cdot B_m^N, \quad (17)$$

$$A \equiv -1 / \left\{ \frac{\partial \Psi(R^*, \psi(R^*, \mu^m, \mu^f))}{\partial R} + \frac{\partial \Psi}{\partial \psi} \cdot \frac{\partial \psi(R^*, \mu^m, \mu^f)}{\partial R} \right\},$$

$$B_m^N \equiv \frac{\partial \psi}{\partial \mu_i^m}.$$

As stated, we treat  $A$  and  $B$  as constants. The second part of (14) shows

$$A > 0.$$

In fact, this  $A$  is common to each of the family–work–and–retirement patterns in this paper. Moving to the sign of the second term,  $B$ , note that (11) implies

$$\frac{\partial y_{it}^m}{\partial \mu_i^m} = y_{it}^m \quad \text{and} \quad \frac{\partial Y_{it}^m}{\partial \mu_i^m} = Y_{it}^m.$$

Similarly for females. Then

$$B_m^N = \frac{\partial \psi}{\partial \mu^m} = \frac{y_R^m}{y_R^m + y_R^f} - \frac{Y_R^m}{Y_R^m + Y_R^f} = \frac{y_R^m}{y_R^m} - \frac{Y_R^m}{Y_R^m} = 1 - 1 = 0.$$

Hence, under Formulation I we expect

$$\alpha_m^N = 0. \tag{18}$$

In words, if only the husband ever works, Formulation I predicts the absence of any relationship between husband’s earning ability and timing of his retirement — the income and substitution effects of higher earning ability on labor supply just cancel one another.

If the wife works at all, we think of retirement together as the base case. We assume further that the ratio  $y_{it}^m/y_{it}^f$  is roughly constant all ages  $t$  in this case. Then letting  $A$  and  $B$  be as in (17),  $A$  is unchanged from above and  $\alpha_m = A \cdot B_m$  with

$$B_m = \frac{y_R^m}{y_R^m + y_R^f} - \frac{Y_R^m}{Y_R^m + Y_R^f} = \frac{y_R^m}{y_R^m + y_R^f} - \frac{y_R^m}{y_R^m + y_R^f} = 0.$$

The relationship is symmetric for female earnings; so, for households in set O Formulation I predicts

$$\alpha_m = 0 = \alpha_f. \tag{19}$$

If the wife works in some years but retires before her husband,  $\alpha_m^S = A \cdot A_m^S$  with

$$B_m^S = \frac{y_R^m}{y_R^m + y_R^f} - \frac{Y_R^m}{Y_R^m + Y_R^f} = \frac{y_R^m}{y_R^m} - \frac{Y_R^m}{Y_R^m + Y_R^f} = 1 - \frac{Y_R^m}{Y_R^m + Y_R^f} = \frac{Y_R^f}{Y_R^m + Y_R^f} > 0.$$

For  $\alpha_f^S$ ,

$$B_f^S = \frac{y_R^f}{y_R^m + y_R^f} - \frac{Y_R^f}{Y_R^m + Y_R^f} = 0 - \frac{Y_R^f}{Y_R^m + Y_R^f} = -\frac{Y_R^f}{Y_R^m + Y_R^f} < 0.$$

Hence, for households in set S we expect

$$\alpha_m^S > 0, \quad \alpha_f^S < 0, \quad \alpha_m^S + \alpha_f^S = 0. \tag{20}$$

This paper’s idea is to estimate (16) and then to derive tests from (18)-(20).

**Formulation II.** Our second formulation of the life-cycle model assumes intratemporal additivity of consumption expenditure and leisure. To generate such a formulation, assume that for some  $\gamma < 1$  and  $\Gamma > 0$  we have

$$u(c, t, R) \equiv \begin{cases} \frac{1}{\gamma} \cdot [c]^\gamma, & \text{if } t < R, \\ \frac{1}{\gamma} \cdot [c]^\gamma + \Gamma, & \text{if } t \geq R. \end{cases} \quad (21)$$

It is straightforward to adapt our previous analysis to Formulation II. Expression (21) determines flow utility; (5)-(6) remain valid if we set  $\lambda = 1$  (implying  $\Lambda = 1$  as well); the new free-endpoint condition is

$$[c_{R-}]^{\gamma-1} \cdot y_{R-} = \Gamma \iff \frac{y_{R-}}{[c_{R-}]^{1-\gamma}} = \Gamma; \quad (22)$$

the replacement for (10) is

$$\begin{aligned} \ln\left(\frac{y_{R-}}{[c_{R-}]^{1-\gamma}}\right) &= \ln(y_{R-}/Y(R)) + \gamma \cdot \ln(Y(R)) - (1-\gamma) \cdot R \cdot \sigma \\ &+ (1-\gamma) \cdot \ln\left(\int_0^T e^{-r \cdot s} \cdot e^{\sigma \cdot s} ds\right); \end{aligned} \quad (23)$$

and, we replace (12) with

$$\psi(R, \mu^m, \mu^f) \equiv \ln\left(\frac{y_R^m + y_R^f}{Y_R^m + Y_R^f}\right) + \gamma \cdot (Y_R^m + Y_R^f). \quad (24)$$

Our use of the implicit function theorem, Taylor approximation, the general form of (17), and the form of  $A$  with  $A > 0$  remain the same as in Formulation I. Again, different family work-and-retirement scenarios lead to different expressions for  $B$  and thus different expressions for the alpha coefficients in (16).

In the existing literature, almost all estimates or calibrations of  $\gamma$  have  $\gamma \leq 0$ , and we accordingly assume  $\gamma < 0$ . In fact, the great majority of existing estimates have  $\gamma < -1$ .

As with Formulation I, we derive parameter constraints case by case.

For households in set N,

$$B_m^N = \frac{\partial \psi}{\partial \mu^m} = \frac{y_R^m}{y_R^m + y_R^f} - \frac{Y_R^m}{Y_R^m + Y_R^f} + \gamma \cdot \frac{Y_R^m}{Y_R^m + Y_R^f} = \frac{y_R^m}{y_R^m} - \frac{Y_R^m}{Y_R^m} + \gamma \cdot \frac{Y_R^m}{Y_R^m} = \gamma.$$

Hence, we expect

$$\alpha_m^N < 0. \quad (25)$$

If both the wife and husband work until  $R$ , and if their earnings are proportional to one another at all ages,

$$B_m = \frac{\partial \psi}{\partial \mu^m} = \frac{y_R^m}{y_R^m + y_R^f} - \frac{Y_R^m}{Y_R^m + Y_R^f} + \gamma \cdot \frac{Y_R^m}{Y_R^m + Y_R^f} = \gamma \cdot \frac{Y_R^m}{Y_R^m + Y_R^f},$$

$$B_f = \frac{\partial \psi}{\partial \mu^f} = \frac{y_R^f}{y_R^m + y_R^f} - \frac{Y_R^f}{Y_R^m + Y_R^f} + \gamma \cdot \frac{Y_R^f}{Y_R^m + Y_R^f} = \gamma \cdot \frac{Y_R^f}{Y_R^m + Y_R^f}.$$

Hence, for households in set O we expect

$$\alpha_m < \alpha_f < 0 \quad \text{and} \quad \alpha_m^N = \alpha_m + \alpha_f. \quad (26)$$

If the wife works but retires prior to  $R$ , we have

$$B_m^S = \frac{\partial \psi}{\partial \mu^m} = \frac{y_R^m}{y_R^m + y_R^f} - \frac{Y_R^m}{Y_R^m + Y_R^f} + \gamma \cdot \frac{Y_R^m}{Y_R^m + Y_R^f} =$$

$$\frac{y_R^m}{y_R^m} - \frac{Y_R^m}{Y_R^m + Y_R^f} + \gamma \cdot \frac{Y_R^m}{Y_R^m + Y_R^f} = \frac{Y_R^f}{Y_R^m + Y_R^f} + \gamma \cdot \frac{Y_R^m}{Y_R^m + Y_R^f},$$

$$B_f^S = \frac{\partial \psi}{\partial \mu^f} = \frac{y_R^f}{y_R^m + y_R^f} - \frac{Y_R^f}{Y_R^m + Y_R^f} + \gamma \cdot \frac{Y_R^f}{Y_R^m + Y_R^f} = -\frac{Y_R^f}{Y_R^m + Y_R^f} + \gamma \cdot \frac{Y_R^f}{Y_R^m + Y_R^f}.$$

Hence, for households in set S we expect

$$\alpha_m^S + \alpha_f^S = \alpha_m + \alpha_f, \quad \alpha_m < \alpha_m^S < 0, \quad \alpha_f^S < \alpha_f < 0. \quad (27)$$

We estimate (16) and then develop tests of Formulation II from (25)-(27).

**Discussion** A careful examination of the preceding sections shows that despite superficial dissimilarities, Formulations I-II are indistinguishable when  $\gamma = 0$  (in which case both are additively separable). Hope for empirically distinguishing between them rests, therefore, with having  $\gamma$  distinctly different from 0.

**Generalization of Formulation I.** Auerbach and Kotlikoff [1987] use a CES function for service flows from consumption expenditures and leisure:

$$f(c, \ell) \equiv [c^\omega + \alpha \cdot \ell^\omega]^{1/\omega}, \quad \omega \equiv 1 - \frac{1}{\Omega}, \quad \Omega > 0, \quad (28)$$

where  $\Omega$  is the elasticity of substitution. Setting  $\Omega = 1$ , we have our Formulation I. The interesting alternative has  $\Omega \in (0, 1)$ .

Unfortunately, the algebra for CES formulation is formidable. Deriving numerical signs, say, for each of Auerbach and Kotlikoff's parameter values, would be possible. Intuitively, it seems likely that an elasticity of substitution less than 1 would induce higher earning households to retire earlier (as in the case of Formulation II).

### 3. Empirical Analysis

This paper’s goal is to employ cross-sectional data to test predictions from the two life-cycle formulations discussed above.

**Data.** The central elements for our empirical analysis are male and female earnings equations (11) and retirement equation (16). We use data from the original HRS sample 1992-2002. The survey data enables us to determine couples’ retirement ages. It also provides information on education, number of children, marital history, birth dates, and earnings. Restricted data from the Social Security system provides earnings histories back to 1951. One problem is that the latter are right censored: prior to 1980, the histories only report earnings up to the Social Security Tax cap; after 1980, they disguise earnings \$125,000-250000, \$250000-500000, and \$500000+. A second censoring problem arises with retirement ages: not all HRS men have retired by 2002, and some who have retired did so because of disability. Dropping censored households does not seem a good option since it would drastically reduce the sample size and leave a very selected sample. Our estimation strategy attempts to deal with both censoring problems.

To best match the assumptions of our framework, we restrict attention to married couples neither partner of which was ever married to a different spouse. Other restrictions include spouse age difference less than or equal to 6 years, no more than 6 children, both spouses having 9-24 years of schooling, males having linked Social Security earning histories and positive years of employment, women either never worked prior to 1992 or having linked Social Security earning histories, women having never worked prior to 1951, and men retiring in the age range 52-71.<sup>4</sup>

In fact, we work with two subsamples. One consists of households with males have 12 years of schooling. The other has males with 16-17 years of schooling. Table 1 provides details.

**Likelihood Functions.** The data censoring described above complicates our estimation procedure a great deal. Consider first the male and female earnings equations from (11). Let the residuals be

$$e_{it}^m(\beta^m) \equiv \ln(y_{it}^m) - X_{it}^m \cdot \beta^m \quad \text{and} \quad e_{it}^f(\beta^f) \equiv \ln(y_{it}^f) - X_{it}^f \cdot \beta^f,$$

where  $y_{it}^m$ , for instance, is actual male earnings at experience  $s$  if a non-censored observation is available, and it is the censoring bound otherwise. Let  $\phi(x, \sigma^2)$  be the density for a normal random variable with mean 0; let  $\Phi(x, \sigma^2)$  be the distribution function for the same. Let ages  $s$  have uncensored earnings and  $t$  have censored earnings. Then the negative log likelihood function for male earnings is

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<sup>4</sup> We assume that an individual starts work at age  $\max\{18, \text{years of education} + 6\}$ , that men work full time until they retire, and that women only work in years for which we have earning records. The last requires that we exclude women who could have worked in years before 1951 (when the detailed Social Security earning histories begin).

$$\begin{aligned}
-\ln(\mathcal{L}^m) &= \\
& - \sum_i \ln \left( \int_{-\infty}^{\infty} \prod_s \phi(e_{is}^m(\beta^m) - \mu_i^m; \sigma_{\epsilon^m}^2) \cdot \right. \\
& \quad \left. \prod_t [1 - \Phi(e_{it}^m(\beta^m) - \mu_i^m; \sigma_{\epsilon^m}^2)] \cdot \phi(\mu_i^m, \sigma_{\mu^m}^2) d\mu^m \right). \tag{29}
\end{aligned}$$

The female earnings likelihood is analogous. To estimate the covariance of  $\mu^m$  and  $\mu^f$ , let ages  $s'$  have uncensored female earnings and ages  $t'$  have censored female earnings. Then

$$\begin{aligned}
-\ln(\mathcal{L}^{mf}) &= \\
& - \sum_i \ln \left( \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \prod_s \phi(e_{is}^m(\beta^m) - \mu_i^m; \sigma_{\epsilon^m}^2) \cdot \prod_t [1 - \Phi(e_{it}^m(\beta^m) - \mu_i^m; \sigma_{\epsilon^m}^2)] \cdot \right. \\
& \quad \prod_{s'} \phi(e_{is'}^f(\beta^f) - \mu_i^f; \sigma_{\epsilon^f}^2) \cdot \prod_{t'} [1 - \Phi(e_{it'}^f(\beta^f) - \mu_i^f; \sigma_{\epsilon^f}^2)] \cdot \\
& \quad \left. \phi((\mu_i^m, \mu_i^f), \Sigma_{\mu^m, \mu^f}^2) d\mu^m d\mu^f \right). \tag{30}
\end{aligned}$$

To estimate (16), we need all three regression equations together. Using the notation of (16), let

$$\beta^r \equiv (\xi, \alpha_m^N, \alpha_m, \alpha_f, \alpha_m^S, \alpha_f^S),$$

$$\begin{aligned}
e_i^r(\beta^r, \mu_i^m, \mu_i^f) &\equiv R_i - \\
& \{Z_i \cdot \xi + \mu_i^m \cdot \chi_i^N \cdot \alpha_m^N + \mu_i^m \cdot \chi_i \cdot \alpha_m + \mu_i^f \cdot \chi_i \cdot \alpha_f + \mu_i^m \cdot \chi_i^S \cdot \alpha_m^S + \mu_i^f \cdot \chi_i^S \cdot \alpha_f^S\},
\end{aligned}$$

where  $R_i$  is the actual male retirement age if the latter is observable, and it is the last observable age of work otherwise. Let households with an uncensored male retirement age be indexed with  $i$  and those with censored male retirement age be indexed with  $j$ . Then our likelihood function is

$$\begin{aligned}
-\ln(\mathcal{L}^{mfr}) &= \\
& - \sum_i \ln \left( \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \prod_s \phi(e_{is}^m(\beta^m) - \mu_i^m; \sigma_{\epsilon^m}^2) \cdot \prod_t [1 - \Phi(e_{it}^m(\beta^m) - \mu_i^m; \sigma_{\epsilon^m}^2)] \cdot \right. \\
& \quad \prod_{s'} \phi(e_{is'}^f(\beta^f) - \mu_i^f; \sigma_{\epsilon^f}^2) \cdot \prod_{t'} [1 - \Phi(e_{it'}^f(\beta^f) - \mu_i^f; \sigma_{\epsilon^f}^2)] \cdot
\end{aligned}$$

$$\begin{aligned}
& \phi(e_i^r(\beta^r, \mu_i^m, \mu_i^f); \sigma_\eta^2) \cdot \phi((\mu_i^m, \mu_i^f), \Sigma_{\mu^m, \mu^f}) d\mu_i^m d\mu_i^f) - \\
& \sum_j \ln \left( \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \prod_s \phi(e_{js}^m(\beta^m) - \mu_j^m; \sigma_{\epsilon^m}^2) \cdot \prod_t [1 - \Phi(e_{jt}^m(\beta^m) - \mu_j^m; \sigma_{\epsilon^m}^2)] \cdot \right. \\
& \quad \prod_{s'} \phi(e_{js'}^f(\beta^f) - \mu_j^f; \sigma_{\epsilon^f}^2) \cdot \prod_{t'} [1 - \Phi(e_{jt'}^f(\beta^f) - \mu_j^f; \sigma_{\epsilon^f}^2)] \cdot \\
& \quad \left. [1 - \Phi(e_j^r(\beta^r, \mu_j^m, \mu_j^f); \sigma_\eta^2)] \cdot \phi((\mu_j^m, \mu_j^f), \Sigma_{\mu^m, \mu^f}) d\mu_j^m d\mu_j^f \right). \tag{31}
\end{aligned}$$

For future reference, let the vector of parameters to be estimated in (31) be  $\delta$ :

$$\begin{aligned}
\delta &\equiv (\delta^m, \delta^f, \delta^\rho, \delta^r), \quad \delta^m \equiv (\beta^m, h_{\epsilon^m}), \quad \delta^f \equiv (\beta^f, h_{\epsilon^f}), \quad \delta^\rho \equiv \rho, \\
\delta^r &\equiv (\xi, \alpha_m^N, \alpha_m, \alpha_f, \alpha_m^S, \alpha_f^S, h_\eta), \tag{32}
\end{aligned}$$

with  $h_{\epsilon^m}$  the precision  $1/\sigma_{\epsilon^m}$ , etc., and  $\rho$  the correlation coefficient for  $\mu^m$  and  $\mu^f$ .

**Estimation.** The complexity of likelihood function (31) leads us to a multi-stage estimation strategy. (i) We estimate earnings dynamics equations for men and their spouses for each subsample. See Table 2. Each earning dynamics equation includes a quadratic in experience and a spline of time dummies (designed to capture the effect of technological progress). We employ likelihood (29) in each case. Minimization of (29) yields consistent parameter and standard error estimates. We deflate earnings using the NIPA PCE deflator. As described above, many of the earnings figures are right censored — and our likelihood function takes this into account. The likelihood function has an individual effect,  $\mu^m$  and  $\mu^f$  for men and women, respectively, as well as an iid error. (ii) Fixing parameter values from stage (i), we estimate the correlation coefficient for  $\mu^m$  and  $\mu^f$  from (30). This yields a consistent parameter estimate (though not, in the conventional output, a standard error). Fixing estimates of  $(\delta^m, \delta^f, \delta^\rho)$ , we minimize (31) with respect to  $\delta^r$  — see (32). Again, this develops consistent parameter estimates  $\hat{\delta}^r$ . (iii) Using our consistent parameter estimates as starting values, we then take a single step of Newton’s method, with respect to all parameters of the model (i.e, all elements of  $\delta$ ), using (31). This so-called “one-step MLE” procedure yields asymptotically efficient parameter estimates and consistent standard errors for all parameters. We repeat the procedure for both subsamples.

Table 3 presents our one-step MLE results — which are the next section’s topic.

## 4. Results

We estimate the full model in two ways. The first allows 5 components in the vector  $\alpha$  for the retirement regression:

$$\alpha \equiv (\alpha_m^N, \alpha_m, \alpha_f, \alpha_m^S, \alpha_f^S).$$

As one might imagine, the estimates sometimes agree, and sometimes disagree, with the text’s *a priori* constraints on these parameters — and many of the component parameters

are imprecisely estimated. For any confidence level  $x$ , using our point estimate  $\hat{\alpha}$  and its covariance matrix we can construct a confidence ellipsoid, say,  $E(x)$ . *A priori* constraints from the text limit the domain for  $\alpha$  consistent with Formulation I to

$$F^I \equiv \{\alpha \mid \alpha_m^N = 0, \alpha_m = 0, \alpha_f = 0, \alpha_m^S \geq 0, \alpha_m^S + \alpha_f^S = 1\}.$$

If the data reject Formulation I, we should see  $F^I \cap E(x) = \emptyset$ . In practice, however, given the imprecision of our estimates, even for  $x = 0.50$  we can find points in  $F^I \cap E(x)$ . Similarly, Formulation II yields a set

$$F^{II} \equiv \{\alpha \mid \alpha_m^N \leq 0, \alpha_m \leq \alpha_f \leq 0, \alpha_m^N = \alpha_m + \alpha_f, \\ \alpha_m^S + \alpha_f^S = \alpha_m^N, \alpha_m \leq \alpha_m^S \leq 0, \alpha_f^S \leq \alpha_f \leq 0\}.$$

Again the intersection of sets, in this case  $F^{II} \cap E(x)$ , is non-empty even for  $x = 0.50$ . In other words, unrestricted estimates are not close to yielding a rejection of either Formulation I or II.

Rather than pursuing unrestricted estimation further, we re-parameterize as follows. As above, partition each subsample into a sets N, S, and O. We estimate a two-component version of  $\alpha$ ,  $\alpha = (\alpha_0, \alpha_1)$ . Set  $Y_R^m / (Y_R^m + Y_R^f)$  and  $Y_R^f / (Y_R^m + Y_R^f)$  to their subsample averages ( $Y_R^f / (Y_R^m + Y_R^f) = 0.1488$  for high school males and  $0.1328$  for college males). For households in N, let the relevant segment of the retirement Tobit in (31) be

$$\alpha_1 \cdot \mu^m; \tag{33}$$

for set O, let the segment be

$$\alpha_1 \cdot \left( \frac{Y_R^m}{Y_R^m + Y_R^f} \cdot \mu^m + \frac{Y_R^f}{Y_R^m + Y_R^f} \cdot \mu^f \right); \tag{34}$$

and, for set S, let it be

$$\alpha_1 \cdot \left( \frac{Y_R^m}{Y_R^m + Y_R^f} \cdot \mu^m + \frac{Y_R^f}{Y_R^m + Y_R^f} \cdot \mu^f \right) + \alpha_0 \cdot \left( \frac{Y_R^f}{Y_R^m + Y_R^f} \cdot (\mu^m - \mu^f) \right). \tag{35}$$

Then we precisely capture the text's *a priori* constraints for Formulation II if and only if

$$\alpha_0 = A \quad \text{and} \quad \alpha_1 = \gamma \cdot A.$$

And, we capture Formulation I if and only if

$$\alpha_0 = A \quad \text{and} \quad \alpha_1 = 0.$$

Table 3 reports one-step MLE results given re-parameterization (33)-(35).

The isoelastic utility function is very widely used in the economics literature and virtually all estimates (and calibrations) of  $\gamma$  are negative — generally with  $\gamma < -1$ . As we have seen, second-order conditions imply  $A > 0$ . Thus, we expect

$\alpha_0 > 0$  and  $\alpha_1 < 0$  for Formulation II.

Furthermore,

$$\alpha_1/\alpha_0 = \gamma \quad \text{under Formulation II.} \tag{36}$$

In contrast, we expect

$$\alpha_0 > 0 \quad \text{and} \quad \alpha_1 = 0 \quad \text{under Formulation I.}$$

(Notice that the ratio of (36) does not determine  $\gamma$  under Formulation I.)

Table 3 shows that the subsample of college-educated men generally supports Formulation I: our estimate  $\widehat{\alpha}_0 > 0$ , and we are not close to rejecting  $\alpha_1 = 0$ . The subsample of high school-educated men also supports Formulation I in the sense that the estimate  $\widehat{\alpha}_1$  is not significantly different from 0. But, the negative point estimate for  $\alpha_0$  in this case is disconcerting (though its sign is not statistically significant).

## 5. Conclusions

A tentative conclusion is that Formulation I is supported by our cross-sectional data. Nevertheless, more precise estimates, especially for  $\alpha_0$ , would be welcome. In the near future, we will, accordingly, seek to add HRS 2004 data, to derive full (rather than one-step) maximum likelihood estimates, and to check the role of additional HRS information about retirement along the lines of Brown [2002] — especially for high school graduates.

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Table 1A: Sample Inclusion Details  
 Married HRS Couples, Male with 16-17 years Education  
 [number households]

MARRIED	4227
MARRIED ONCE ONLY	2734
MEETS ED CRITERIA	670
SP AGE DIFF LE 6	649
Number KIDS LE 6	649
M BIRTHYR 1926-43	634
M SSA LINK and WORKED	487
F SSA LINK OR NVER WK92	447
M USABLE RET DATA	447
F WORKED BUT NOT PRIOR 1951	431
M RET AGE [52, 71]	398

Table 1B: Sample Inclusion Details  
 Married HRS Couples, Male with 12 years Education  
 [number households]

MARRIED	4227
MARRIED ONCE ONLY	2734
MEETS ED CRITERIA	878
SP AGE DIFF LE 6	848
Number KIDS LE 6	848
M BIRTHYR 1926-43	831
M SSA LINK and WORKED	616
F SSA LINK OR NVER WK92	566
M USABLE RET DATA	564
F WORKED BUT NOT PRIOR 1951	510
M RET AGE [52, 71]	479

Table 2A: MLE Earnings Dynamics Model  
 Male with 16-17 years Education  
 [MALE: 398 households; 8571 observations;  
 FEMALE: 350 households; 4350 observations]

	MEN		WOMEN	
CONSTANT	9.445895	S.E.= 0.042922	9.303197	S.E.= 0.045870
EXP	0.037957	S.E.= 0.005436	0.023942	S.E.= 0.003722
EXP**2/100	-0.082559	S.E.= 0.011095	0.003456	S.E.= 0.009986
DUM 51-60	0.027645	S.E.= 0.013767	-0.026086	S.E.= 0.015128
DUM 61-65	0.055045	S.E.= 0.011377	0.030682	S.E.= 0.014535
DUM 66-70	0.068264	S.E.= 0.011262	0.023457	S.E.= 0.012579
DUM 71-75	0.017483	S.E.= 0.009391	-0.040677	S.E.= 0.010275
DUM 76-80	-0.053238	S.E.= 0.006858	0.001592	S.E.= 0.007429
DUM 81-85	0.026366	S.E.= 0.005625	0.008759	S.E.= 0.006129
DUM 86-90	0.008231	S.E.= 0.005642	-0.014425	S.E.= 0.005849
DUM 91-95	0.032732	S.E.= 0.007932	0.007393	S.E.= 0.007094
DUM 96-2002	0.007534	S.E.= 0.012495	0.028155	S.E.= 0.008338
H E	2.211134	S.E.= 0.022673	2.540731	S.E.= 0.029668
H U	2.396899	S.E.= 0.094700	2.816944	S.E.= 0.127405

Table 2B: MLE Earnings Dynamics Model  
 Male with 12 years Education  
 [MALE: 479 households; 12966 observations;  
 FEMALE: 424 households; 5618 observations]

	MEN		WOMEN	
CONSTANT	9.178359	S.E.= 0.027014	9.010817	S.E.= 0.034493
EXP	0.037744	S.E.= 0.003272	0.026888	S.E.= 0.002706
EXP**2/100	-0.067587	S.E.= 0.006143	-0.019585	S.E.= 0.007225
DUM 51-60	0.002233	S.E.= 0.005600	0.018959	S.E.= 0.010401
DUM 61-65	0.044253	S.E.= 0.005391	-0.011222	S.E.= 0.009055
DUM 66-70	0.063514	S.E.= 0.005656	0.019070	S.E.= 0.007608
DUM 71-75	0.001947	S.E.= 0.005086	-0.027299	S.E.= 0.006424
DUM 76-80	-0.024495	S.E.= 0.004388	0.007314	S.E.= 0.005433
DUM 81-85	0.000440	S.E.= 0.004099	-0.006472	S.E.= 0.004520
DUM 86-90	-0.012110	S.E.= 0.004221	-0.007137	S.E.= 0.004394
DUM 91-95	0.010977	S.E.= 0.006191	-0.002036	S.E.= 0.005340
DUM 96-2002	0.051630	S.E.= 0.011480	0.016760	S.E.= 0.006547
H E	2.526538	S.E.= 0.019463	3.003165	S.E.= 0.030098
H U	3.425625	S.E.= 0.123969	3.616138	S.E.= 0.148334

Table 3: One-Step MLE Complete Model

	Male 16-17 Years Education Sample: 398 households	Male 12 Years Education Sample: 497 households
<b>MALE EARN:</b>		
CONSTANT	9.431033 S.E.= 0.061685	9.191908 S.E.= 0.026464
EXP	0.037069 S.E.= 0.003344	0.037653 S.E.= 0.001509
EXP**2/100	-0.081269 S.E.= 0.006616	-0.066924 S.E.= 0.002886
DUM 51-60	0.029594 S.E.= 0.013421	0.000378 S.E.= 0.004193
DUM 61-65	0.054341 S.E.= 0.008542	0.045100 S.E.= 0.004230
DUM 66-70	0.070509 S.E.= 0.008453	0.063022 S.E.= 0.004622
DUM 71-75	0.016524 S.E.= 0.006919	0.002272 S.E.= 0.003971
DUM 76-80	-0.051993 S.E.= 0.005498	-0.024442 S.E.= 0.003700
DUM 81-85	0.026671 S.E.= 0.005250	-0.000475 S.E.= 0.003518
DUM 86-90	0.008660 S.E.= 0.004704	-0.012917 S.E.= 0.003589
DUM 91-95	0.032122 S.E.= 0.006721	0.011979 S.E.= 0.005664
DUM 96-2002	0.008032 S.E.= 0.010849	0.051129 S.E.= 0.010661
H E	2.210703 S.E.= 0.007674	2.529225 S.E.= 0.007463
H U	2.201327 S.E.= 0.116044	3.290666 S.E.= 0.164056
<b>FEMALE EARN:</b>		
F CONSTANT	9.396634 S.E.= 0.078180	8.998625 S.E.= 0.025278
F EXP	0.023766 S.E.= 0.001254	0.024816 S.E.= 0.001140
F EXP**2/100	0.005333 S.E.= 0.004248	-0.014492 S.E.= 0.003140
F DUM 51-60	-0.047344 S.E.= 0.019095	0.024832 S.E.= 0.007381
F DUM 61-65	0.036192 S.E.= 0.006905	-0.011481 S.E.= 0.005223
F DUM 66-70	0.018098 S.E.= 0.006562	0.015332 S.E.= 0.004246
F DUM 71-75	-0.041194 S.E.= 0.005771	-0.020339 S.E.= 0.003453
F DUM 76-80	0.003467 S.E.= 0.004147	0.002119 S.E.= 0.003047
F DUM 81-85	0.006693 S.E.= 0.003754	-0.002758 S.E.= 0.002728
F DUM 86-90	-0.013625 S.E.= 0.003188	-0.008540 S.E.= 0.002128
F DUM 91-95	0.006920 S.E.= 0.005786	-0.004470 S.E.= 0.002771
FDUM 96-2002	0.028893 S.E.= 0.006292	0.019524 S.E.= 0.003672
F H E	2.541738 S.E.= 0.006258	3.009319 S.E.= 0.011143
F H U	3.308950 S.E.= 0.218487	3.934467 S.E.= 0.239697
<b>MALE/FEMALE CORRELATION:</b>		
RHO	-0.034411 S.E.= 0.060097	-0.106717 S.E.= 0.061654

Table 3: One-Step MLE Complete Model (cont.)

	Male 16-17 Years Education Sample: 398 households	Male 12 Years Education Sample: 497 households
RETIREMENT TOBIT [MALE]:		
T CONSTANT	59.563864 S.E.= 4.457244	63.597939 S.E.= 3.915067
T RACE 1 BK	-1.925862 S.E.= 1.634227	1.040268 S.E.= 1.158164
T SPAGE DIFF	0.228462 S.E.= 0.121087	0.088047 S.E.= 0.112395
T SELF EMPL	2.832953 S.E.= 0.700414	1.701303 S.E.= 0.617534
M B DUM26-28	0.884260 S.E.= 1.732592	-1.104802 S.E.= 1.389326
M B DUM29-31	-0.117463 S.E.= 0.647443	0.817340 S.E.= 0.468577
M B DUM32-34	-0.088882 S.E.= 0.478326	-0.540441 S.E.= 0.362423
M B DUM35-37	0.048368 S.E.= 0.371205	0.644533 S.E.= 0.306405
M B DUM38-40	0.310847 S.E.= 0.411782	-0.395191 S.E.= 0.367513
M B DUM41-43	-0.239885 S.E.= 1.003217	2.904733 S.E.= 1.185669
TOBIT H E	0.195678 S.E.= 0.011373	0.223717 S.E.= 0.009590
ALPHA0	8.475418 S.E.= 11.230247	-6.264984 S.E.= 11.325800
ALPHA1	-0.346817 S.E.= 0.900188	-1.096992 S.E.= 1.008304
ADDENDUM:		
GAMMA	-0.040920	0.175099